Interactive comment on “Multi-periodic climate dynamics: spectral analysis of long-term instrumental and proxy temperature records” by H.-J. Lüdecke et al.

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Suboptimal spectrum estimation methods used by Lüdecke et al.

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Interesting manuscript, mainly through the interpretation with Feigenbaum's work, which should be further elaborated.

The major problem, however, is that the basis for the interpretation, namely the spectral analysis (Section 3), has been performed poorly. The reported periods of spectral peaks (Figure 3 left/right) are meaningless.

I take the liberty to comment just on Section 3 on spectrum estimation and the DFA (which serves spectrum estimation by providing a noise alternative). The intention is to make the authors (and the reviewers as well) aware of points where the paper needs to be strengthened. A final remark is on reproducibility via making data available and describing methods on an algorithmic level.

Spectrum is a property of a stochastic random process (for which further the Fourier transform of its autocovariance function has to exist) (e.g., Priestley, 1981). Deterministic, systematic, long-term components are not random; they are therefore removed from the data series (and interpreted) prior to the spectral analysis (e.g., Mudelsee, 2010). Unremoved trends may appear as low-frequency, spurious peaks in the estimated spectrum (e.g., Schulz and Mudelsee, 2002). The presented manuscript,
which analyses temperature during the instrumental period (and before), fails to con-
sider trends and trend removal, despite the fact that we have at least one systemat-
ic, long-term component influencing temperature, namely atmospheric concentrations of
greenhouse gases (Solomon et al., 2007). Why does the manuscript ignore trends?

Spectrum estimation 2. DFT.
The authors are correct that the Discrete Fourier Transform (DFT) is a simple method
(p. 4496, l. 18). However, it has poor properties for estimating a spectrum (Priestley,
1981; Mudelsee, 2010). Remember that we wish to estimate a whole function (the
spectrum) on basis of a finite-size and noise-influenced time series. The estimate is
not exactly equal to the true, but unknown spectrum: the estimate exhibits bias and
variance. For example, the DFT-estimated spectra (Figure 3) have, even if the pro-
cess is a harmonic process (pure sinusoidal components), errors of about 200% on
the frequency borders (zero, Nyquist) and 100% elsewhere (Priestley, 1981, Section
6.1.3 therein). That means: any interpretation of spectral peaks in Figure 3 is obso-
lete. The authors are advised to use better (although less simple) methods of spec-
trum estimation, namely Thomson’s multitapers in case of evenly spaced series and
the Lomb–Scargle periodogram with Welch’s Overlapped Segment Averaging (WOSA)
procedure in case of unevenly spaced series (Mudelsee, 2010, Chapter 5 therein).

Spectrum estimation 3. Interpretation of low-frequency peaks.
As said, trends (or remnants of trends when not completely removed) may appear as
spurious low-frequency peaks in a spectrum estimate. A guideline therefore is not to
consider too low frequencies (Mudelsee, 2010); at least two full cycles should fall into
the time span covered by the time series. In case of the 254-year M6 series, one
cannot say anything meaningful about periods longer than about 127 years; referring
to other, longer series from somewhere else does not help.

Since the time span covered by the series is finite, the resolution of the estimated spec-
trum is larger than zero, and spectral peaks are not sharp but have a width, described
by a parameter (6-dB bandwidth). This quantity is a must when reporting spectrum
estimation results, it helps readers to assess the accuracy of results.

Spectrum estimation 5. AR(1) alternative.
Hasselmann (1976) gave a convincing theoretical derivation why AR(1) red noise is a
suitable null hypothesis of climate ("trace of random weather noise"). The manuscript
(p. 4497, l. 26) should not say that this is "unrealistic." I would rather recommend
to show both noise alternatives (AR(1) and long memory) in Figure 3. This would
strengthen the reliability of resulting peaks (which have to jump above each of
the noise-alternative hurdles).

It is indeed interesting to consider this as an additional alternative—if enough data
points are available. However, M6 is too short. Re-invoking the monthly data (p. 4498,
l. 6ff) is problematic since this is another scale (monthly) than on what the spectrum
estimate is based (yearly). Furthermore, DFA is not the optimum method for estimat-
ing the long-memory parameter (see Mudelsee (2010, Chapter 2 therein) and refer-
ces cited therein). It is unfortunate that physicists tend to like to ignore that better
long-memory estimation methods (i.e., better bias/variance properties, better robust-
ness), such as ARFIMA models combined with maximum likelihood (Beran, 1994), are
available. It is further mandatory to report estimation errors also for the long-memory
parameter. Without this, the claim that the result (p. 4498, l. 9) of $\alpha = 0.58$" demon-
strates the unsuitability of the AR(1) alternative (which has $\alpha = 0.50$), is not justified.

Final remark on reproducibility: data.
To allow readers and reviewers to reproduce the results, the raw data described in
Section 2 (8 x instrumental temperature, 1 x ice core, 1 x stalagmite) and also the data products (M6; standardized series) have to be made available for download.

**Final remark on reproducibility: methods description.**

To allow readers and reviewers to reproduce the results, the used methods have to be described on an algorithmic level. Bad examples from the manuscript are the following. The DFA method is almost completely undescribed (detrending method, scaling range, fit method for determining the power-law or long-range parameter, uncertainty of parameter estimation). Surrogate records with long memory: it is insufficient to cite a whole book (Turcotte, 1997 [reference: see manuscript]) without detailing the page numbers where the method is described; the type of long-memory process (fractional Gaussian noise, ARFIMA, other?) is not mentioned. Bandwidth of spectrum estimation is not communicated. Analogously for the wavelet analysis: no significance test is performed, no scale bar is given in Figure 4, no sensitivity experiment on the effects of interpolation of wavelet estimation is reported.—Some work to be done!

**References**


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