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Climate of the Past Discussions



# On the differences between two semi-empirical sea-level models for the last two millennia

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## Abstract

We compare hindcasts of global mean sea level over the past millennium obtained using two semi-empirical models linking temperature and sea-level rise. The models differ in that one of them includes a term for a very long-term sea-level rise component unfolding over many millennia. On short (century) time scales, both models give very similar results.

Proxy sea-level reconstructions from the northern (North Carolina) and southern (New Zealand and Tasmania) hemispheres are used to test the ability of both models to reproduce the longer-term sea-level evolution. In both comparisons the model including the second term produces a markedly better fit from 1000 AD to the present.

When both models are used for generating sea-level projections, they behave similarly out to 2100 AD. Further out, to 2300–2500 AD, the projections differ significantly, in no small part due to different values for the sea-level response time scale $\tau$ obtained. We conclude that careful model validation on long time scales is important before attempting multi-century projections.

## 1 Introduction

Sea-level rise is a serious consequence of ongoing climate change, and its confident projection into the remainder of this century (and beyond) is important for mitigating and managing risk in the coastal zone (Church et al., 2010). Unfortunately, models based on physical processes are not yet mature enough to make accurate and robust projections, because dynamic systems such as ice sheets are insufficiently understood (IPCC, 2007). For this reason, semi-empirical approaches to modelling and projecting global sea-level rise are used as an alternative (e.g. Rahmstorf, 2007; Grinsted et al., 2010; Vermeer and Rahmstorf, 2009; Rahmstorf et al., 2011; Jevrejeva et al., 2011; Kemp et al., 2011; Schaeffer et al., 2012). This approach seeks a simple relationship between a bulk quantity that physics-based models project reliably (global mean



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surface temperature) and the quantity of interest (sea-level rise). The models are calibrated over a time period for which data for both quantities are available. In most cases this is the instrumental period with measurements of temperature and sea level, but proxy data are used for calibration on longer time scales (Kemp et al., 2011). The relationship developed over this calibration period then forecasts sea-level rise under scenarios of future changes in global mean temperature. The central assumption is that the relationship found for the calibration period remains valid into the future.

A critical issue for semi-empirical and process-based models is validation with performance assessed by how well they reproduce past sea-level changes. In this paper we investigate differences between two recent semi-empirical models (Grinsted et al., 2010, henceforth called G10, and Kemp et al., 2011, henceforth called K11) that made sea-level hindcasts for the last millennia from proxy-reconstructed temperatures. The two models are characterized mathematically and through numerical calculations to analyse similarities and differences between them. We identify a term representing a very long response time scale as the key difference, allowing the K11 model to better fit high resolution proxy sea-level reconstructions from North Carolina, USA, New Zealand and Tasmania (Kemp et al., 2009, 2011; Gehrels et al., 2008, 2012). We also show how both models behave when projecting sea level several centuries into the future. The model producing a generally better fit to historical sea-level data makes significantly higher sea-level projections; the reasons for this are explained.

## 2 Model formulations

Both semi-empirical models (G10 and K11) assume that sea-level rise is proportional to warming above some baseline climate. For example, a step-function warming by amount $\Delta T$ causes a rate of sea-level rise proportional to $\Delta T$, which decays exponentially with a decay time scale $\tau$. In G10, $\tau$ is the only time scale in the model, so over time, the rate of rise decays to zero and sea-level reaches a new equilibrium, higher than the initial sea level by an amount proportional to $\Delta T$. K11 includes an additional contribution, a long-term or "perpetual" rate of sea-level rise, to reflect a sea-level response on a time scale much longer than that of model applicability. Therefore the K11 model never reaches an equilibrium sea level and does not attempt to describe an equilibrium response. Rather, K11 approximates the transient sea-level response over a limited time interval. Depending on the time scale of interest, different terms are required for this. This is conceptually a series expansion of sea-level response terms on progressively longer time scales. The $b$-term introduced by Vermeer and Rahmstorf (2009) represents the shortest time scales (e.g. ocean mixed layer response, this term is not considered in this paper), the $a$-term of Rahmstorf (2007) is the linear response in the rate of rise up to one or two centuries, while the multi-century time scale $\tau$ needs to be explicitly resolved to extend applicability of the model up to one or two millennia.

The G10 model has the form:

$$\frac{dS}{dt} = \frac{1}{\tau}\left(S_{eq}(t) - S(t)\right), \tag{1}$$

where equilibrium sea level for a given temperature $T$ is

$$S_{eq}(t) = S_{eq}(T(t)) = AT(t) + B. \tag{2}$$

Model parameters to be estimated are $\tau$, $A$, $B$, and an integration constant called $S_0$ (sea level $S$ for the first time epoch of every temperature time series). Jevrejeva et al. (2011) used the same model to make sea-level projections out to 2500 AD, but with radiative forcings taking the role of the independent variable, given above to global mean temperature $T$.

A conceptual problem with this formulation, if taken too literally, is that the initial rate of sea-level rise in response to a temperature change $\Delta T$ is

$$\frac{dS}{dt} = \frac{\Delta S_{eq}}{\tau},$$

where (for illustration) we start from a previous equilibrium state, and $\Delta S_{eq}$ is the change in equilibrium sea level due to the temperature change. The *initial rate* is thus





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directly proportional to the *final equilibrium rise* due to a given warming. This is unintuitive because a sea-level change that will not occur until far into the future is used to explain contemporary sea-level rise. It may also be physically wrong. For thermal expansion, the final equilibrium rise will depend on the total ocean depth, but the initial rate of ocean heat uptake (and thus sea-level rise) is determined by conditions near the ocean surface and is insensitive to ocean depth. For ice sheets warming just above a threshold for complete melting would add the same amount of mass to the ocean (the final magnitude of equilibrium rise) as a scenario where the warming was twice as large. The difference would only be the rate of sea-level rise. The final equilibrium rise is governed by different physical processes and properties than the initial rate of rise. Final equilibrium is governed by bulk properties like total ocean depth and total ice mass, while the rates of sea-level rise for the next century or so are governed by flux properties, such as the rates of heat uptake by ice masses and ocean waters. The problems caused by assuming proportionality between these two fundamentally different things is discussed.

In K11 the following model was used, a modification of those in Rahmstorf (2007, henceforth called R07) and Vermeer and Rahmstorf (2009, henceforth called VR09):

$$\frac{\mathrm{d}S}{\mathrm{d}t} = a_1\left(T(t) - T_{0,0}\right) + a_2\left(T(t) - T_0(t)\right)\left[+ b\frac{\mathrm{d}T}{\mathrm{d}t}\right], \tag{3}$$

$$\frac{\mathrm{d}T_0}{\mathrm{d}t} = \frac{1}{\tau}\left(T(t) - T_0(t)\right). \tag{4}$$

An initial value for $T_0$ must be given for the start of the model computation. Further model parameters are $\tau$, $a_1$, $T_{0,0}$, $a_2$, $b$, and a sea-level integration constant (an arbitrary offset presenting sea level relative to a chosen baseline). The time scale $\tau$ corresponds to the same parameter in the G10 model.

K11 used the tide-gauge data sea-level fit from VR09 as a prior constraint; here, we instead used an uninformative prior. Then, on the millennial time scale, the rapid-response term $b\,\frac{\mathrm{d}T}{\mathrm{d}t}$ in Eq. (3) can be almost ignored and we do so for clarity.





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Both models are formally equivalent, except for the additional "perpetual" term in K11. For proof, define $T_0(t)$ by:

$$S(t) = S(T_0(t)) = AT_0(t) + B.$$

Substituting this and Eq. (2) into Eq. (1) produces

$$\frac{dS}{dt} = \frac{A}{\tau}(T(t) - T_0(t)). \tag{5}$$

Identifying

$$a = A/\tau$$

shows equivalence to Eq. (3) without the $a_1$ and $b$ terms:

$$\frac{dS}{dt} = a(T(t) - T_0(t)).$$

Furthermore,

$$\frac{dS}{dt} = A\frac{dT_0}{dt}$$

substituted into Eq. (5) shows that $T_0(t)$ satisfies Eq. (4).

It readily follows that the ratio between the sea level and temperature disequilibria is $A$:

$$S_{eq}(t) - S(t) = A(T(t) - T_0(t)).$$

In the G10 model, the proportionality factor between temperature $T(t)$ and sea-level rise $\frac{dS}{dt}$ equals $A/\tau$, whereas in K11 it equals $a = a_1 + a_2$. This factor, which we call sea-level sensitivity, is of particular importance, since it determines how fast sea level





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rises initially after a given warming. Typical values range between 3 and 6 $\mathrm{mm\,yr^{-1}\,K^{-1}}$ (where larger values tend to correspond to shorter $\tau$), so that a step-function global warming by 1 °C leads to an initial rise between 3 and 6 $\mathrm{mm\,yr^{-1}}$. This is consistent with the ratio between the increase in global mean temperature of 0.8 °C since pre-industrial times and the observed 3.2 $\mathrm{mm\,yr^{-1}}$ sea-level rise of recent decades (Rahmstorf et al., 2011).

In the G10 model sea-level sensitivity is again proportional to the factor $A$ which determines how strongly the *equilibrium* sea level depends on temperature. For example when $a = 6$ and $\tau = 200$ yr the factor $A = 1.2\,\mathrm{m\,K^{-1}}$, while for $a = 3$ and $\tau = 1000$ yr, $A = 3\,\mathrm{m\,K^{-1}}$. We will come back to this point later.

## 3 Calibration and data

For calibration (i.e. to determine the model parameters), both papers use Bayesian inference with stochastic simulation ("Monte Carlo"). An ensemble is generated containing a large number of individual temperature and sea-level scenarios based on parameters drawn at random from their simulated prior distributions.

G10 used the instrumental period to develop their semi-empirical model. The global tide-gauge compilation of Jevrejeva et al. (2008) provided the sea level input. This record has a yearly value for global sea level since 1700 AD, but prior to 1850 AD is necessarily reliant on a handful of discontinuous gauges situated exclusively in north-western Europe: Amsterdam, since 1700 AD, Liverpool, since 1768 AD, and Stockholm, since 1774 AD. Additionally G10 used sea level reconstructed from Roman fish tanks in Italy (Lambeck et al., 2004) as a constraint for sea level in 0 AD. For temperature input, G10 used the proxy reconstructions of Moberg et al. (2005) and of Jones and Mann (2004), the former being their preferred one.

Kemp et al. (2011) used a longer period of time for model calibration by employing a proxy based reconstruction of sea level to extend beyond the instrumental period. They reconstructed sea level since 200 BC at two sites in North Carolina, USA (NC) using



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foraminifera preserved in coastal salt-marsh sedimentary deposits. Dating of sediment samples using $^{14}C$ in conjunction with other methods and age-depth models produced a high-resolution (decadal and decimeter scale) sea-level reconstruction. For temperature input, K11 used the global mean surface temperature record in Mann et al. (2008, henceforth called M08) derived with the error-in-variables (EiV) method. This data set spans the period from 500 to 2006 AD.

## 4 Differences between local and global mean sea level

Both models assume that the sea-level time series used as an input are representative of global mean sea level. However, this correspondence is only approximate because the tide gauges that make up the Jevrejeva et al. (2008) record are not evenly distributed (particularly in the period prior to 1850 AD) and the NC sea-level reconstruction is from a single part of the Atlantic Ocean. There are three principal sources of spatial variability that cause sea level at any location to deviate from the global mean:

1. Vertical land movements (uplift or subsidence) primarily associated with glacial isostatic adjustment (GIA) of the solid Earth (Clark et al., 1978). Tectonic motion influences relative sea level, but sites are typically selected from areas of assumed stability.

2. The change in the Earth's gravity field when land-based ice melts, leading to a geographically uneven distribution of the added ocean water volume (Mitrovica et al., 2001).

3. Dynamic sea surface topography effects caused by prevailing currents and winds and ocean density (steric) changes (Marsh et al., 1990).

GIA typically causes the largest deviations for a specific location from the global mean sea-level rate because its acts in a single direction and can have regional rates of up to several mm/yr. Therefore comparison among sea-level records from different regions





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or approximation of global mean sea level must be preceded by a correction for the estimated contribution of GIA. Over the last 2000 yr, GIA for any location not close to present-day glaciations may be assumed to be a linear rate because of the response time of the solid Earth (Peltier, 2004).

The individual tide-gauge records that make up the global compilation used by G10 were adjusted for GIA using values estimated by an Earth-Ice model (ICE 5-G, Peltier, 2004). These models are constrained in part by proxy-based reconstructions of sea level (e.g. Fairbanks, 1989) and assume that all changes since the end of meltwater input are a consequence solely of GIA. The timing of when meltwater input ceased is a point of discussion, but estimates are typically between 2000 and 4000 yr BP. Predictions from Earth-Ice models for the US Atlantic coast systematically misfit proxy sea-level reconstructions, with the difference attributed to mantle heterogeneity and/or tectonic contributions (Engelhart et al., 2009, 2011).

An alternative means to estimate the contribution of GIA is to use regional sea-level reconstructions making the plausible and widely accepted assumption that all sea-level changes the last 2000 yr were caused only by vertical land motion because total ocean volume did not change appreciably (e.g. Gehrels et al., 2011, 2012). This approach therefore accounts for both GIA and tectonic effects whilst the use of field-based data circumvents some of the limitations of Earth-Ice models (which share the basic premise). However, this approach can only be employed where adequate (spatial and temporal coverage free from compaction) reconstructions exist. K11 used this geological approach for the NC sea-level reconstruction. They used data from 2000 yr BP to 1900 AD as the estimate of vertical land motion as presented and described in Engelhart et al. (2009, 2011). The pattern of subsidence along the US Atlantic coast is that expected of a collapsing glacial forebulge and the rate used for NC is consistent with neighboring regions (see Figs. 2 and 3 in Engelhart et al., 2009). The K11 model is able to accommodate any estimated GIA rate without loss of fit if the rate is constant.

Due to inherent uncertainty in estimating the GIA contribution, all sea-level records (instrumental or proxy reconstruction) are *approximations* of global mean sea level. In



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G10 the mean trend from the three long tide-gauge records was assumed to equal the global trend for the 18th and 19th century within ± 6 cm. K11 suggested that after correction for GIA, the NC proxy reconstruction approximated average global sea level for the past two millennia within ± 10 cm. By comparing 20th century sea-level change in the tide-gauge record (Jevrejeva et al., 2008) and North Carolina reconstruction (Kemp et al., 2011) the validity of these assumptions can be considered. K11 showed 24 ± 5 cm of sea-level rise during the 20th century compared to ∼ 19 cm for Jevrejeva et al. (2008). Thus within their stated uncertainties the sea-level data sets used as inputs for G10 and K11 describe the same 20th century sea-level change. Further, the characteristic period of stable sea level in the tide-gauge record between 1700 AD and 1850 AD is faithfully reproduced by the North Carolina reconstruction.

## 5 The discrepancy before 1100 AD

Data typically become sparser and of poorer quality further back in time and are accompanied by less understanding of the system to be modelled. This circumstance leads to a *hierarchy of time scales*, where model performance is characterized by how far back in time the model manages to faithfully describe the data. In K11 the sea-level hindcast agrees well with the proxy reconstruction back to about 1100 AD, before which it diverges.

We propose three possible explanations for this discrepancy:

1. *Reconstructed temperatures in M08 before 1100 AD are too high by 0.2 K.*
   K11 showed that a downward adjustment in the proxy temperature by 0.2 K before 1100 AD entirely removes the discrepancy between observed and modelled sea level. Given that we used a global temperature reconstruction, which especially in the Southern Hemisphere and over the oceans is based on only few data before 1100 AD, such a small temperature bias is a plausible and parsimonious explanation. An adjustment of 0.2 K is within the range of variation seen among reconstruction variants presented in M08.

2. *The semi-empirical model parameters that apply before and after 1100 AD are not the same.*
   This explanation is in its practical effects the same as adjusting the M08 temperature reconstruction. It would suffice, e.g. to assume that the parameter value $T_{0,0}$ before would be higher than after. Semi-empirical models are approximations of a complex system in terms of "bulk" quantities. Expanding the body of data to which a given model is exposed will inevitably make it perform poorer at the task of reconciling all that data. The K11 model may well be too simple for more than 1000 yr of data.

3. *The North Carolina sea-level reconstruction exceeds (by up to 0.5 m at 500 AD) and does not approximate global sea level before 1100 AD*.
   However, proxy data from the Atlantic (Donnelly et al., 2004) and Gulf (González and Törnqvist, 2009) coasts of the United States, Southern Cook Islands (Goodwin and Harvey, 2008) Mediterranean Sea (Lambeck et al., 2004; Sivan et al., 2004), and Iceland (Gehrels et al., 2006) show stable rather than rising sea-levels in the first millennium (Kemp et al., 2011). The IPCC AR4 stated that "the average rate of sea-level rise over the last 2 kyr was zero and at most in the range 0 to 0.2 mm $yr^{-1}$".

Evidence to directly support or refute these explanations is thin, underscoring the importance of obtaining a sufficient number of geographically diverse proxy temperature and sea-level reconstructions. We focus on modelling sea level for only the last millennium, for which temperature and sea-level proxy data of reasonable quantity and quality are currently available.

## 6 Numerical experiments

To better understand the different behaviour of the G10 and K11 model hindcasts, and consequently the expected behaviour in projection calculations, we performed the following computational experiments.

1. We gained insight using the G10 model in a simple deterministic fashion, performing a number of integrations over time of the model Eqs. (1) and (2). Model parameter values were tuned manually. Firstly, we successfully replicated the Bayesian hindcast result presented in G10. Then we attempted to find the best fit, still using the G10 model, to the North Carolina sea-level proxy reconstruction from K11. In both experiments we also examined the effect of using different temperature proxy time series.

2. To better understand the impact of the K11 being fitted to a sea-level proxy reconstruction from a single geographical location (North Carolina), we undertook a similar study using alternative sea-level proxy data from the other side of the globe, New Zealand and Tasmania.

3. To understand why the G10 model produces a characteristic "hump" around 1000 AD in its sea-level hindcasts, we hindcast sea level using the K11 sea-level proxy data, but restricted to the period post-1700 AD (like the tide-gauge data in G10). We replicated the hump by modifying the algorithm of K11 to become formally equivalent to that of G10.

4. We analyzed, using an idealized temperature scenario for 2000–2500 AD, the difference in sea-level projections between the G10 and K11 model formulations. This is relevant as actual projections based on more realistic temperature or radiative-forcing scenarios, out to 2500 AD (Jevrejeva et al., 2011) and 2300 AD (Schaeffer et al., 2012) have recently been published.





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## 7 Results

### 7.1 Characterizing the differences between G10 and K11

In Eq. (1), if $T(t)$ is roughly constant over time, then $S(t)$ will appoach $S_{eq}(T)$ in an exponential decay fashion on a time scale of $\tau$. For time spans several times $\tau$ or longer (or, equivalently, for $\tau$ much shorter than the time span studied) $S(t)$ will be approximately constant over time. More loosely, if $T(t)$ does not include any long term trend, then neither will $S(t)$, meaning that a constant global temperature is associated with stable sea level.

In contrast, Eq. (3) shows that, *even if* $T(t)$ *is constant*, $S(t)$ can still have a trend:

$$\left\langle \frac{dS}{dt} \right\rangle = a_1 \left( \langle T \rangle - T_{0,0} \right), \tag{6}$$

unless $T_{0,0} = \langle T \rangle$. Here, the angle brackets denote average over time.

This term is approximately a linear trend in sea level over 1000–1900 AD because $T(t) - T_{0,0}$ is approximately constant over this interval (for a plausible value $a_1 \sim 0.1$ cm yr$^{-1}$ K$^{-1}$ we have $T_{0,0} = -0.6$ K below the 1400–1800 AD level, see Sect. 7.4). Given that this term is formally the only difference between the G10 and K11 models on longer time scales, we examine if this term is needed or whether the data can be described without it. To do so requires data over multiple centuries, since the difference between the two models vanishes on time scales shorter than $\tau$ because the terms with $a_1$ and $a$ in Eq. (3) are not distinct in this case.

### 7.2 Replicating the Grinsted et al. result

Hindcasts plotted as simple curves in Figs. 1 and 2 result, where not stated otherwise, from integration of the ordinary differential Eqs. (1) and (2), using manually tuned model parameters. This approach avoids the complexities of Bayesian inference and shows that the behaviour of the hindcasts with a particular temperature dataset follows directly from the mathematical form of the model.

To ensure correctness of code we replicated the original G10 hindcast by manually tuning the model parameters, keeping $\tau = 208$ yr, and integrating. We thus obtained one consistent scenario, whereas the original hindcast consists of epoch-wise ensemble median values. This exercise was repeated (also with $\tau = 208$ yr) by replacing the temperature data originally used by G10 (Moberg et al., 2005), with the M08 dataset that K11 employed (magenta curve in Fig. 1). This substitution of temperature datasets created more substantial differences before 1100 AD.

Agreement for the later years and disagreement for the earlier years suggests that the temperature proxy data contain more long term uncertainty before 1100 AD.

Figure 1 also shows the alternative NC sea-level reconstruction curve if we assume GIA subsidence to be 1.3 mm yr$^{-1}$ as proposed in Grinsted et al. (2011), rather than 1.0 mm yr$^{-1}$. Note that, contrary to Eqs. (1) and (2), the K11 semi-empirical model formulation of Eqs. (3) and (4) by its nature is able to accomodate any value of GIA subsidence with the same quality of fit seen in Fig. 2.

### 7.3 Fitting to the Kemp et al. sea-level data

We tested how well the semi-empirical models can reproduce the North Carolina proxy-based sea-level reconstruction from 1100 AD to the present (Fig. 2). The K11 model constrained by these proxy data shows that $\tau$ is likely shorter than 1000 yr, and that $-a_1 T_{0,0}$ (representative of the long response time "perpetual" contribution to sea-level rise) is very likely greater than zero.

For reference we also include the fits, already presented in K11, to the NC sea-level reconstruction using the model versions of Rahmstorf (2007) and Vermeer and Rahmstorf (2009) in the graph, although these models explicitly were not intended for multi-century time scales. The relevant semi-empirical model equations are:



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$$\frac{\mathrm{d}S}{\mathrm{d}t} = a\left(T(t) - T_0\right),$$
$$\frac{\mathrm{d}S}{\mathrm{d}t} = a\left(T(t) - T_0\right) + b\frac{\mathrm{d}T}{\mathrm{d}t},$$

for R07 and VR09, respectively, where parameters $a$ and $b$ were retained from the fit to instrumental data (Church and White, 2006) but $T_0$ was adjusted upward because it is constrained by the period of near-stable sea level 1400–1800 AD. The R07 and VR09 models calibrated over the instrumental period (1880–2000 AD), lack a several-century response time scale, and are therefore equivalent to the G10 model *with a very large* $\tau$ (several kyrs at least). These models produce decent though sub-optimal fits, because K11 demonstrated that the likelihood for very large $\tau$ values is small (Fig. 3, panel c). Therefore only solutions with time-varying $T_0(t)$, yielding suitable $T_0$ values both for the instrumental and the pre-instrumental periods, are somewhat likely.

We also constrained the G10 model using the NC sea-level reconstruction but with *all* its parameters freely adjustable, using the Moberg et al. (2005) and M08 temperature datasets. Only after setting $\tau$ to a very large value (we chose 4000 yr) did further manual parameter tuning produce reasonable fits, although again less optimal than the K11 hindcast. The lack of fit between the G10 model and NC sea-level reconstruction unless a response time of several thousand years is used provides circumstantial evidence that the NC data contain such a long response time component.

In the G10 model, short values for $\tau$ cannot produce a reasonable fit to the NC sea-level reconstruction. Instead, a sea-level "hump" around 1000 AD appears, similar to that in G10. This is because for small $\tau$, the effect of initial sea level $S_0$ vanishes asymptotically after a few centuries. Equilibrium sea level $S_{eq}$ again is fixed to true sea level for temperatures from 1700–1800 AD, known from observations and proxies. Then, given $\tau$, $A$ is fixed by the ratio between the modern temperature and sea-level upswings, and $B$ follows. This completely determines the sea-level curve for 1000 AD and later, and illustrates how tightly constrained the degrees of freedom of the model are by a few aspects of the data.





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## 7.4 Perpetual sea-level rise

We start by analyzing the magnitude of both terms in the simulation since 1000 AD in K11. Equation (6) gives the perpetual rate of sea-level rise *under long-term constant temperatures* (meaning long compared to the explicitly resolved multi-century time scale $\tau$). This perpetual rate of rise could be related to a sea-level response on the multi-millennial time scale of large ice sheets and/or GIA, which is an ongoing process today.

In reality, temperature was not constant but slowly decreased, possibly since the mid-Holocene optimum (Rohde, 2005). The equation that applies (derived by substituting Eq. 4 into Eq. 3) is

$$\begin{aligned}\left\langle \frac{dS}{dt} \right\rangle &= a_1 \left(\langle T \rangle - T_{0,0}\right) + a_2 \tau \left\langle \frac{dT_0}{dt} \right\rangle \\ &= a_1 \left(\langle T \rangle - T_{0,0}\right) + a\tau \left\langle \frac{dT_0'}{dt} \right\rangle .\end{aligned}$$

Here, in the second term,

$$T_0'(t) = \frac{1}{a}\left(a_1 T_{0,0} + a_2 T_0(t)\right).$$

We may read off $\left\langle \frac{dT_0'}{dt} \right\rangle$ for the period since 1100 AD from the "equilibrium temperature" curve in Fig. 4c in K11, yielding $-0.15\,\mathrm{K\,kyr^{-1}}$. With $\tau \approx 400\,\mathrm{yr}$ and $a = 0.56\,\mathrm{cm\,yr^{-1}\,K^{-1}}$ we obtain $-0.3\,\mathrm{mm\,yr^{-1}}$ for the second term. With the first term being $0.6 \pm 0.3\,\mathrm{mm\,yr^{-1}}$ (Kemp et al., 2011), it follows that long-term global sea ocean volume (taken to be the same as global mean sea level) did not significantly deviate from constancy (Gehrels et al., 2011, 2012; Milne et al., 2009); see however Gehrels (2010).

One may short-circuit the above derivation by observing that

$$\left\langle \frac{dS}{dt} \right\rangle = \frac{S(t_2) - S(t_1)}{t_2 - t_1},$$





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and applying this to the K11 sea-level proxy reconstruction for, e.g. $t_1 = 120\,\mathrm{BC}$ and $t_2 = 1800\,\mathrm{AD}$, leaving the recent upswing out of consideration. Any long-term net average sea-level rise is insignificant. The earlier derivation shows us that total sea-level rise is statistically indistinguishable from zero, but that it consists of two distinct and opposite components that individually are significantly different from zero. There is a very long-term (intermillennial) rise that may be a remnant from the last deglaciation, and a long-term fall produced by the time scale $\tau$ (intercentennial) response to the downward trend of late Holocene temperature.

A further indication that such a long response time scale is missing from the G10 model is given by the magnitude of equilibrium sea level changes implied by this model, typically between 1 and $3\,\mathrm{m\,K^{-1}}$ as we already mentioned above. This is small when compared to paleoclimatic estimates of sea-level sensitivity to temperature changes. This is best illustrated by past warm greenhouse climates like that of the Mid-Pliocene, three million years ago. Estimates for sea level range from 10 to 40 m higher than today (Raymo et al., 2011), while global-mean temperature was just 2–3 °C warmer (IPCC, 2007), which would imply a slope of about $3–20\,\mathrm{m\,K^{-1}}$. Until the Antarctic Ice Sheet started to form ~34 million years ago, Earth was practically ice-free for tens of millions of years, implying ~70 m higher sea-level, but proxy data suggest 5–12 °C warmer global temperatures (e.g. Zachos et al., 2001, implying a slope of about $6–14\,\mathrm{m\,K^{-1}}$. Even with the substantial uncertainties in the paleoclimatic data, this suggests that the G10 model does not capture a major component of sea-level change on long time scales.

Data from recent interglacials also point to larger sea-level changes than implied by the G10 model (Rohling et al., 2009). However, these are less suited as analogue because they are Milankovich-forced and thus cannot be expected to be proportional to global-mean temperature. For example, for the best-documented last interglacial (LIG) sea-level rise has been estimated as 5.5–9 m above present (Dutton and Lambeck, 2012), but it is unclear if global temperature was cooler or warmer than present (McKay et al., 2011). Arctic summer temperature can have been several degrees warmer while



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global mean temperature was colder than present. Unlike for climate changes caused by global-mean forcing like greenhouse gases or solar variability, for Milankovich-forced changes we do not expect a simple connection between global-mean temperature and sea level.

### 7.5 Bayesian hindcast experiments

Our standard computation uses the Bayesian simulation from K11 as described by Eqs. (3) and (4), and uses as input sea-level proxy data from 1000–2000 AD. We tried out several modifications to this:

1. We forced $a_1$ to zero. This eliminates the first term on the right-hand side of Eq. (3), representing the long time scale or "perpetual" component of sea-level rise. This makes the semi-empirical relationship equivalent to that of G10, as was formally shown above. Furthermore we used the NC sea-level data after 1700 AD only, to create a situation similar to that in G10, where only tide-gauge data after 1700 AD were used. Doing so (Fig. 3, panel a, black and grey) replicated the characteristic G10 hump. The hindcast sea level with its uncertainty bands clearly do *not* accommodate the NC sea-level reconstruction.

2. With $a_1$ forced to zero, using the NC sea level reconstruction after 1100 AD produced no reasonable hindcast, as the likelihoods became tiny. This confirms that the G10 model and the NC reconstruction contradict each other.

3. Using the full K11 model, but only NC sea-level data after 1700 AD (Fig. 3, panel a, black and light blue) shows that this hindcast is compatible with the NC sea level reconstruction on the $1\sigma$ level over the entire past millennium.

One conclusion is that the data from 1700 AD onward are not sufficient to constrain the parameters that determine the longer-term evolution of sea level; a good fit is obtained for 1700–2000 AD with models that differ completely during earlier times. But only the





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K11 model with both terms included provides an uncertainty range for the hindcast that includes the earlier data, i.e. is structurally suited for the task.

## 7.6 Another geographic location

To test the semi-empirical models against proxy sea level data from sites distant to the Atlantic Ocean and in a different geological setting to North Carolina, we used reconstructions from New Zealand (Gehrels et al., 2008) and Tasmania (Gehrels et al., 2012). Both reconstructions were developed from foraminifera preserved in salt-marsh sediment, using an approach that is very similar to K11's reconstruction from North Carolina. These southern hemisphere sites are not affected by ongoing subsidence caused by forebulge collapse (and as a side note, the lack of subsidence may compromise the deposition of these sediments; the Tasmanian data contain a substantial hiatus).

They demonstrate that local sea-level has been stable over the last 2000 yr, although the number of pre-instrumental period data points is small, meaning that the parameters $a_1$ and $\tau$ cannot be meaningfully constrained (Fig. 4, panels b and c). However, $-a_1 T_{0,0}$ ($\sim a_1 \left(T - T_{0,0}\right)$ for the pre-instrumental period) is constrained to positive values, meaning there is a significant long time scale sea-level rise component (panel d).

With the hindcast using our modification which suppresses the long-term component and makes the K11 model equivalent to that of G10, it was harder to obtain a good fit. Furthermore we noticed a *bifurcation*: the histogram plot of $\tau$ showed there to be two distinct peaks, one of values under 400 yr (panel f), and the other of values over 1000 yr (panel g), separated by a range with zero likelihoods. We therefore split the computation into two "branches", for $\tau$ values above and below 750 yr, respectively. The hindcasts themselves look quite different for these branches.

We take this bifurcation to indicate that there are two different response time scales present even in this sparse data.

There are only two useful data points prior to 1800 AD, making the hindcasts non-robust: e.g. no data points exist to constrain the G11 model "hump" around 1000 AD.





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However, these few sea-level reconstruction data points are compatible with the North Carolina reconstruction from opposite hemispheres.

### 7.7 Effect on time scale $\tau$

We examined the effect of suppressing the long-term contribution, by forcing $a_1$ to zero, on the equilibration time scale $\tau$. The standard solution including the $a_1$ term finds $\tau$ unlikely to be over 1000 yr, with preferred values around 400 yr. In contrast, the modified solution with $a_1 = 0$ prefers $\tau$ values in the range 100–200 yr, with values above 400 yr being all but excluded (Fig. 3, panels b and c), which largely agrees with results from Jevrejeva et al. (2011). For the multi-century projections presented in Jevrejeva et al. the predominant smallness of $\tau$ causes a rapid decline in the rate of sea-level rise caused by a given radiative forcing, and rapid equilibration of sea level after forcings are stabilized, at the rather small equilibrium rise discussed earlier. In contrast, Schaeffer et al. (2012), using the K11 model, found much slower equilibration and higher eventual sea level. If the K11 model with its longer response time scale $\tau$ is more realistic, as we have argued here based on paleoclimatic proxy data, then the projections of Jevrejeva et al. (2011) are an underestimate.

For the relative roles of parameters $\tau$ and $a_1$ in projection calculations, cf. Fig. 5 for rough illustration.

## 8 Conclusions

We have compared two semi-empirical models regarding their performance in reproducing the sea-level evolution over the past millennium as reconstructed by proxy data. The two models are equivalent except for an additional “perpetual” term in the K11 model as compared to the G10 model. This term represents time scales longer than those explicitly covered by the response time $\tau$ that is a common parameter of both models. For the past millennium this term may represent a residual small sea-level trend related to the deglaciation. Although the "perpetual" term is small over the pre-industrial period ($\sim 0.6\,\mathrm{mm\,yr^{-1}}$), it has a profound effect on the quality of the solutions on longer time scales. For the period 1700 AD to present, or for projections until 2100 AD, both models obtain very similar results. However, accounting for this perpetual term significantly improves the fit for the last millennium, and it has a major effect on constraining the explicitly resolved time scale $\tau$, with important consequences for multi-century future projections. Due to a suitable combination of circumstances (leaving the perpetual term out, and using only sea-level data after 1700 AD), somewhat counter-intuively model solutions with small $\tau$ ($\sim 200$ yr) are preferred, which then imply small equilibrium sea-level changes at variance with the large paleoclimatic sea-level changes, e.g. in the Mid-Pliocene. Using this approach may cause an underestimation of future sea-level rise on multi-century time scales (Jevrejeva et al., 2011), whereas including the perpetual term leads to higher long-term projections (Schaeffer et al., 2012). Our study highlights the importance of validating semi-empirical sea-level models over longer time scales, and thus the importance of collecting further high-quality millennial sea-level proxy records from different parts of the world.

**Supplementary material related to this article is available online at: http://www.clim-past-discuss.net/8/3551/2012/cpd-8-3551-2012-supplement.zip.**

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**Table 1.** Parameter values used for the integration curves in Figs. 1 and 2.

| Line style | Temperature data | Sea-level target | $A$ (m K) | $B$ (m) | $\tau$ (yr) | $S_0$ (m) |
|---|---|---|---|---|---|---|
| Blue | Moberg et al. (2005) | G10 | 1.3 | 0.77 | 208 | 0.05 |
| Magenta | M08 | G10 | 1.5 | 0.45 | 208 | 0.0 |
| Blue | Moberg et al. (2005) | K11 | 13.6 | 10.5 | 4000 | −1.5 |
| Magenta | M08 | K11 | 20 | 8.6 | 4000 | −1.5 |



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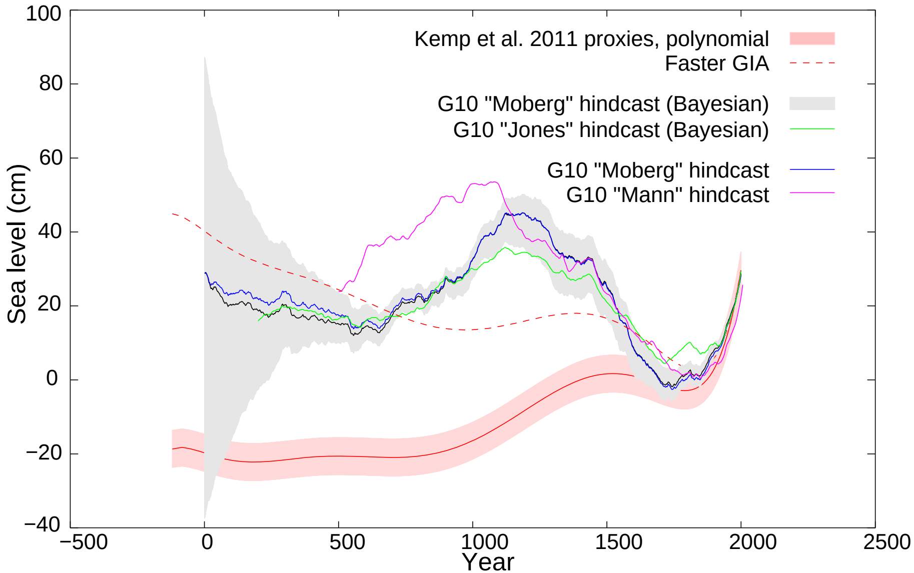


**Fig. 1.** Fitting the G10 semi-empirical model to the North Carolina proxy sea-level data. The original, Bayesian hindcast using the temperature reconstruction from Moberg et al. (2005) is in black and grey. The Bayesian hindcast of G10 using the alternative Jones and Mann 2004 temperature dataset is shown in green. Hindcast by integration of Moberg et al. (2005) following to Eqs. (1) and (2), with manually tuned parameter values, is in blue; and of the M08 temperature reconstruction, in magenta. North Carolina sea-level proxy reconstruction by K11 is summarized by the red and pink curve, which was corrected for GIA using geological data. The dashed red line is the same data with a larger rate of GIA (1.3 mm $yr^{-1}$) removed as proposed by Grinsted et al. (2011). All uncertainty bands are one-sigma.





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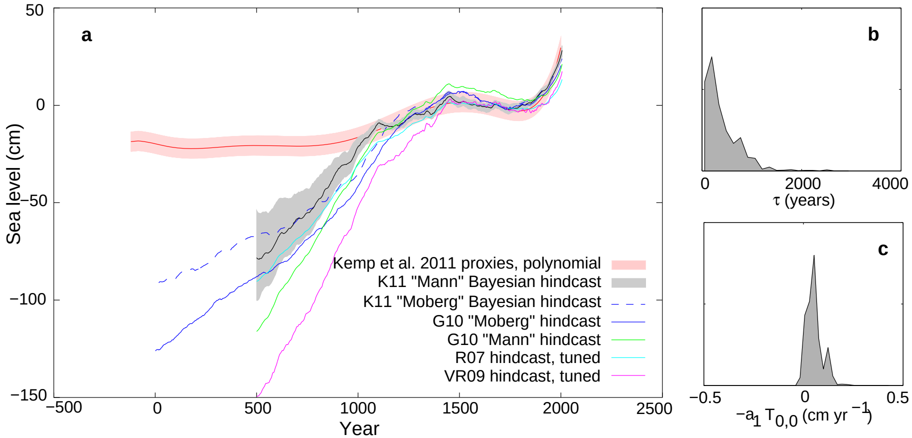


**Fig. 2.** Fitting several semi-empirical models to the North Carolina proxy sea-level data. Panel **(a)**: The Bayesian hindcast using the M08 temperature dataset as presented in K11, using an uninformative prior, is in black and grey; same using Moberg et al. (2005) instead of M08, in dashed blue. Simple hindcast by integration of Moberg et al. (2005) according to Eqs. (1) and (2), with manually tuned parameter values, is in blue; and of the M08 temperature reconstruction, in green. All integration hindcasts have both been manually tuned to fit the North Carolina proxy reconstruction (red, pink) after 1000 AD, with $\tau = 4000$ yr. For comparison are given also the hindcasts from the R07 and VR09 models, with "tuned" values for $T_0$, as in K11. All uncertainty bands are one-sigma. Panel **(b)**: likelihood histogram (arbitrary units) of time scale $\tau$ for the grey hindcast in panel **(a)**. Panel **(c)**: likelihood histogram (arbitrary units) of expression $-a_1 T_{0,0}$ for the grey hindcast in panel **(a)**.



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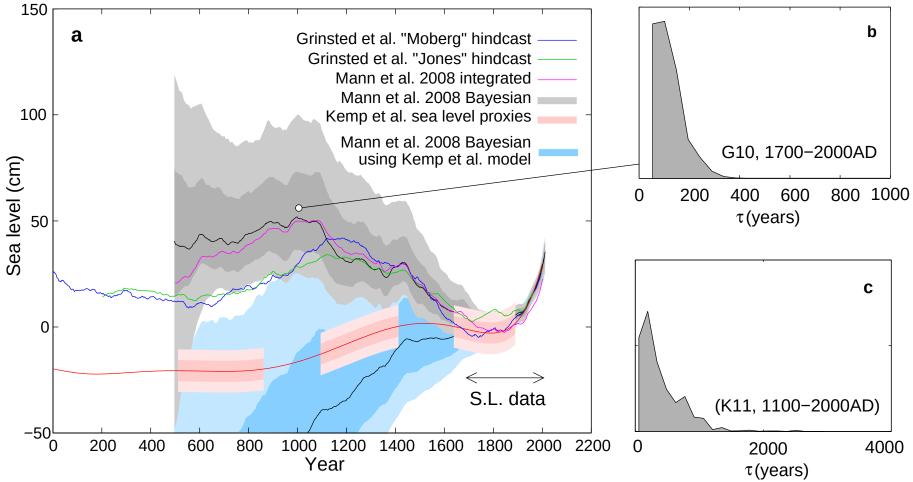


**Fig. 3.** Panel **(a)**: sea-level Bayesian Monte-Carlo hindcast based on reconstructed global temperatures from M08 and reconstructed sea-level values from K11, but only including proxy data after 1700 AD. Uninformative prior. Model used: Eqs. (3) and (4) with $a_1 = 0$ in order to suppress the long time scale component, and $b = 0$. In light blue is given the Bayesian hindcast based on the same data, but using the original model, Eqs. (3) and (4), for comparison. Uncertainty bands correspond to one and two sigma. North Carolina sea-level proxy reconstruction by K11 is summarized by the red and pink curve, which was corrected for GIA using geological data. For comparison, also the G10 “Jones” (green) and “Moberg” (blue) hindcasts are shown, and our own “Mann et al.” hindcast by integration of the G10 equations (magenta) as in Fig. 1. Panel **(b)**: likelihood histogram (arbitrary units) of time scale $\tau$ for the grey hindcast in panel **(a)**. Panel **(c)**: likelihood histogram (arbitrary units) of time scale $\tau$ for the hindcast of K11 with uninformative prior, using data from 1100 AD.

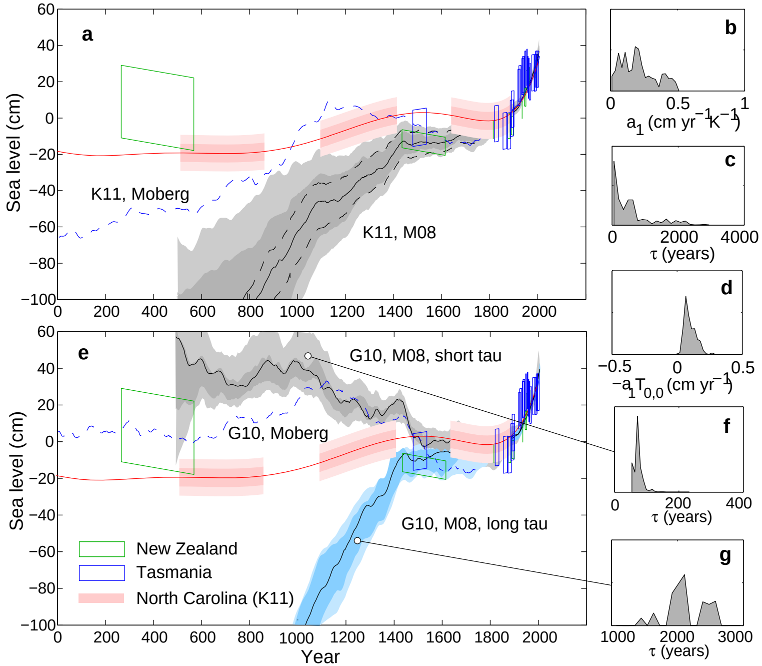


**Fig. 4.** Bayesian hindcasts (black, grey) using the sea-level reconstructions of Tasmania (blue error boxes) and New Zealand (green error boxes) (Gehrels et al., 2012). Hindcasts for high and low GIA values ($\pm 0.15\,\mathrm{mm\,yr^{-1}}$) as black dashed curves. Blue dashed curve denotes the hindcast substituting the Moberg et al. (2005) temperature reconstruction for M08. Panels **(a)**–**(d)**: hindcasts using the K11 model, Eqs. (3) and (4). Panels **(e)**–**(g)**: Hindcasts using the same equations, but with $a_1 = 0$ to suppress the long time scale sea-level rise component, and $b = 0$. Hindcasts for $\tau < 750$ yr and $\tau > 750$ yr separately in grey and light blue. Further items as in previous graphs.





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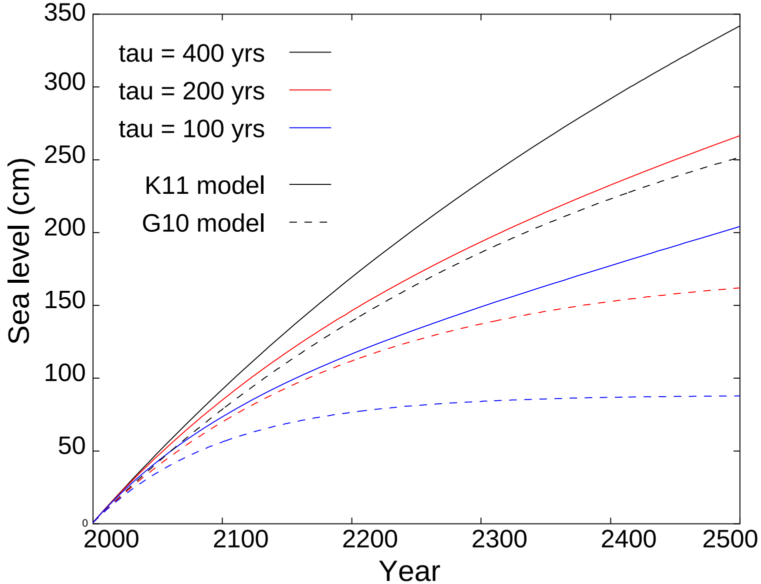


**Fig. 5.** The effects of parameters $\tau$ and $a_1$ in a simple simulated projection computation. Assumed is a step rise in temperature at 2000 AD of +2 K; $a = a_1 + a_2 = 0.43\,\mathrm{cm\,yr^{-1}\,K^{-1}}$; and either $a_1 = 0.1\,\mathrm{cm\,yr^{-1}\,K^{-1}}$ and $T_{0,0} = -0.6\,\mathrm{K}$ (drawn curves), or $a_1 = 0$ (dashed curves).





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