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Abstract

In their paper from 2006, Annan and Hargreaves present an estimate for the uncertainty of climate sensitivity obtained by using Bayes’ theorem to combine information from different sources. In this comment article we criticize two aspects of their reasoning, namely using probability density functions and likelihood functions interchangeably and the assumed independence of evidence from the different sources. The derivation of their result rests on key assumptions, some stated explicitly and some left implicit, which are unlikely to hold in reality. Thus their study does not convincingly reduce the large uncertainty of climate sensitivity remaining in previous observationally-based studies.

1 Introduction

In their work Annan and Hargreaves (Annan and Hargreaves, 2006; hereinafter referred to as AH06) obtain an estimate for the uncertainty of climate sensitivity by combining three different sources of information. The estimate is presented in the form of a posterior (probability) density. In particular, the authors claim having derived the 95% Bayesian confidence (credible) interval (1.7°C, 4.9°C) for the sensitivity parameter. This interval is very narrow in comparison to most observationally-based studies, in which the symmetric 90% confidence interval can contain values of climate sensitivity greater than 10°C (Frame et al., 2005; Hegerl et al., 2007). Annan and Hargreaves use Bayes’ Theorem to combine three sources of observational evidence, viz. estimates from the 20th century warming, volcanic forcing, and the last glacial maximum, into a single estimate of climate sensitivity. We believe that their conclusion is not warranted, as their derivation of the posterior distribution is based on several assumptions, which were left partly implicit, or which, if stated and considered critically, are unlikely to be realistic. The details of our argument are shown below.
2 Bayesian inference

Following the notation in AH06, Bayes’ Theorem can be written in the form

$$f(x|O, H) = f(O|x, H)f(x|H)/f(O|H),$$

(1)

where $x$ = the parameter to be estimated, $H$ = old data, $O$ = new data, and $f$ is a generic notation for conditional probability density functions. Thus this formula expresses how the posterior distribution $f(x|H)$ based on old data $H$ can be updated into the posterior $f(x|O, H)$ by also accounting for the new data $O$ and the likelihood function $f(O|x, H)$. In AH06 the authors apply Bayes’ Theorem on the parameter $x$ = the climate sensitivity, and the data $H$ = 20th century warming, $O$ = ($O_1$, $O_2$) with $O_1$ = volcanic cooling, $O_2$ = last glacial maximum. Then the crucial assumption is made that the three different sources of data, i.e., $H$, $O_1$ and $O_2$, are conditionally independent given the value of the sensitivity parameter $x$. Under this condition, Bayes’ Theorem gets the form

$$f(x|O_1, O_2, H) = f(O_1|x)f(O_2|x)f(x|H)/f(O_1, O_2|H).$$

(2)

Here the denominator does not depend on $x$ and can therefore be viewed as a proportionality constant. It becomes evident from going through the reasoning of AH06 that they use probability density functions (pdfs) and likelihood functions interchangeably, i.e., the likelihood functions $f(O_1|x)$, $f(O_2|x)$ appearing in Bayes’ formula above are replaced by corresponding estimated pdfs $f(x|O_1)$ and $f(x|O_2)$, obtained in AH06 by considering the potential range of values $x$, then choosing three points that they believe would reasonably correspond to the 2.5 and 97.5 quantiles and the mode of $f(x|O_1)$, and similarly of $f(x|O_2)$, and finally fitting Gamma or Gaussian distributions to these points in order to obtain densities. The same method was used for deriving $f(x|H)$.

Two immediate observations can be made concerning this procedure: First, the quantiles and the mode are not based on proper Bayesian posterior densities $f(x|O_1)$, $f(x|O_2)$ and $f(x|H)$ obtained from data by applying the strict rules of Bayesian statistical inference. Second, even if the densities $f(x|O_1)$ and $f(x|O_2)$ had been obtained by
a proper Bayesian analysis and would thereby have the status of posterior density, they
cannot be given the role of likelihood functions $f(O_1|x)$, $f(O_2|x)$ unless a uniform prior
density is assumed in the entire range of $x$. However, in this case it would be a very
strange assumption, particularly in view of the fact that the above formula already in-
volves the prior density $f(x|H)$, and uniform priors should then be assigned twice more
to pdfs that do not originate from Bayesian analyses.

3 Combining data from different sources

In this section we point out a more fundamental problem in the approach of AH06 to
combine the pdfs by the Bayesian method, viz. their assumption that the three differ-
ent sources of data, $H$, $O_1$ and $O_2$, are conditionally independent given the value of
the sensitivity parameter $x$. We believe that this assumption is seriously flawed. To
illustrate this, consider how estimates for climate sensitivity are derived from observa-
tional data. According to the simple heat balance equation (Andreae et al., 2005; Kiehl,
2007):

$$c \frac{d(\Delta T)}{dt} = \Delta Q - \lambda \Delta T,$$

where $c$ is the ocean heat capacity per unit area, $\Delta T$ is the temperature change,
$\Delta Q$ is the total climate forcing due to natural and human factors and $\lambda$ is the
climate feedback parameter. The climate sensitivity is defined as the equilibrium
temperature change due to a doubling of carbon dioxide in the atmosphere, which
means that the relation $\Delta T_{2 \times CO_2} = \Delta Q_{2 \times CO_2}/\lambda$ holds. The radiative forcing resulting from
a doubling of the carbon dioxide concentration level is quite well known and given by
$\Delta Q_{2 \times CO_2} = 3.7 \text{ W/m}^2$ (Ramaswamy et al., 2001). Using the above relation, the climate
sensitivity can be solved:

$$\Delta T_{2 \times CO_2} = 3.7 \frac{\Delta T}{(\Delta Q - c \frac{d(\Delta T)}{dt})}.$$

When estimating the climate sensitivity parameter $x$ from observational data, uncertainty in the values of all three quantities $c$, $\Delta T$ and $\Delta Q$ contribute to the uncertainty in
the derived estimate. Moreover, the latter two, $\Delta T$ and $\Delta Q$, even vary with time. A typical estimate for $c$ is 0.6–1.6 GJ m$^{-2}$K$^{-1}$ (Andreae et al., 2005; Levitus et al., 2000; Folland et al., 2001). Uncertainty in the value of $\Delta Q$ is large as well, the current estimate for anthropogenic $\Delta Q$ by the IPCC being 0.6–2.4 Wm$^{-2}$ (Forster et al., 2007).

The temperature time series data involve various types of uncertainty and noise, which also contributes to the uncertainty of a climate sensitivity estimate. In a proper Bayesian analysis and estimation of climate sensitivity, all these uncertainties in the values of the model parameters should be considered jointly, in terms of their joint probability distributions. The estimate of climate sensitivity would then be obtained by finally integrating out all other parameters from the resulting joint posterior.

Estimates of $c$ and $\Delta Q$ and their uncertainty in the references of AH06 can come, if not from the same source, then at least from similar reasoning and principles. References for all three different observational constraints use climate models in their reasoning (Andronova and Schlesinger, 2001; Knutti et al., 2002; Wigley et al., 2005; Annan et al., 2005) and it is very likely that the different models involve common sources of uncertainty. This concerns particularly estimates of the ocean heat capacity per unit area $c$ but also those of radiative forcing $\Delta Q$. And in fact, the data from volcanic cooling used in AH06 does not include uncertainty of $\Delta Q$ at all (Wigley et al., 2005). Annan and Hargreaves discuss the concern of similar biases in climate models at the end of Sect. 3.4, and exclude two additional lines of evidence from their main conclusion partly because of this, but make no argument as to why the concern would not undermine the conditional independence assumption for the three lines of evidence used.

4 Conclusions

The original idea of AH06, to combine information from different sources and thereby arrive at an improved estimate for the climate sensitivity parameter, is interesting and in principle achievable by applying the tool provided by Bayesian statistical inference. However, the way in which the idea is executed in the paper leaves too many open
questions to be convincing. Annan and Hargreaves are assuming that the climate sensitivity pdfs from different lines of evidence originate from Bayesian analyses using uniform priors and that the different lines of evidence are independent. We have shown that the first assumption does not hold and the latter assumption is also unlikely to hold as the different results may very well have common uncertainties related to radiative forcing and ocean heat capacity. These concerns make the reliability of the resulting pdf highly questionable. Therefore it is our opinion that previous observationally-based studies currently provide us with the best estimates for the uncertainty of climate sensitivity and that high values of climate sensitivity cannot be ruled out in the way done by Annan and Hargreaves.

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References


